error injection. This verifies an assumption in our theory that a relatively coarse grid is sufficient for resolving the unsteady acoustic waves if the steady pressure is accurate.

Conclusion

A simple numerical technique which can be easily implemented in any numerical code for computations of two- or three-dimensional unsteady transonic aerodynamics has been introduced. This technique allows the decoupling of a solution having two distinct length scales into two parts. By solving each part on a grid of the proper length scale, substantial savings in computation time and storage can be achieved.

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New Eddy Viscosity Model for Computation of Swirling Turbulent Flows

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Introduction

THE conventional k-ϵ model has been widely used to predict relatively simple turbulent flows on the grounds of its simplicity and reasonable accuracy. However, it is well known that the model in its standard form is not adequate for taking into account the secondary straining effect of swirl or buoyancy.

According to Bradshaw,[1] the secondary straining increases or decreases the turbulent length scale in the flowfield depending on the stability of the straining field. In order to make the k-ϵ model adaptable to such flow situations, Launder et al.[2] and Rodi[3] have made one of the model constants in their dissipation equations a function of a Richardson number, which will ultimately change the resulting length scale since $\ell \sim k^{3/2}/\epsilon$. However, both of these forms are only ad hoc models and are not well supported by theory. Obviously, the Reynolds stress model does not require additional modification of the model for these flows.[4] But it requires the solution of a number of differential equations for Reynolds stresses, and the computational cost is prohibitively expensive.

This Note is aimed at proposing a simple theoretical modification of the standard k-ϵ model to solve swirling turbulent flows. The coaxial swirling jet of Ribeiro and Whitelaw[5] and the single swirling round jet of Pratte and Keffer[6] are used to test the performance of the proposed method.

Modification of Eddy Viscosity for Swirling Flow

In the standard k-ϵ model, the eddy viscosity ν_t is given by a function of turbulent kinetic energy k and its dissipation rate ϵ,

$$\nu_t = C_\mu (k^2/\epsilon) \tag{1}$$

where the model constant C_μ is taken as a constant value, $C_\mu = 0.09$. The standard transport equations for k and ϵ in cylindrical coordinates are as follows:

$$U\frac{\partial k}{\partial x} + V\frac{\partial k}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial r}\right) + P - \epsilon \tag{2}$$

$$U\frac{\partial \epsilon}{\partial x} + V\frac{\partial \epsilon}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\nu_t}{\sigma_\epsilon}\frac{\partial \epsilon}{\partial r}\right) + C_{\epsilon_1}\frac{\epsilon}{k}P - C_{\epsilon_2}\frac{\epsilon^2}{k} \tag{3}$$

where P is the production rate of k,

$$P = \nu_t\left\{2\left[\left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial r}\right)^2 + \left(\frac{V}{r}\right)^2\right] + \left(\frac{\partial U}{\partial r} + \frac{\partial V}{\partial x}\right)^2 + \left(\frac{\partial W}{\partial x}\right)^2 + \left[r\frac{\partial}{\partial r}\left(\frac{W}{r}\right)\right]^2\right\}$$

and the model constants are $\sigma_k = 1$, $\sigma_\epsilon = 1.22$, $C_{\epsilon 1} = 1.44$, and $C_{\epsilon 2} = 1.92$. The velocity components U, V, and W are in the axial (x), radial (r), and tangential (θ) directions, respectively.

The derivation of our modified eddy viscosity model begins with the following algebraic stress equations[7]:

$$\frac{\overline{u_i u_j}}{k} = \phi_1 \frac{P_{ij}}{\epsilon} + \phi_2 \delta_{ij} \tag{4}$$

where

$$\phi_1 = \frac{1 - c_2}{(P/\epsilon) + c_1 - 1} \quad \text{and} \quad \phi_2 = \frac{2}{3}\frac{c_2(P/\epsilon) + c_1 - 1}{(P/\epsilon) + c_1 - 1}$$

The constants c_1 and c_2 are inertial and forced return-to-isotropy constants respectively.[8] δ_{ij} is a Kronecker delta, and P_{ij} is the production tensor of the Reynolds stresses $\overline{u_i u_j}$

$$P_{ij} = -\overline{u_i u_k}\frac{\partial U_j}{\partial x_k} - \overline{u_j u_k}\frac{\partial U_i}{\partial x_k}$$

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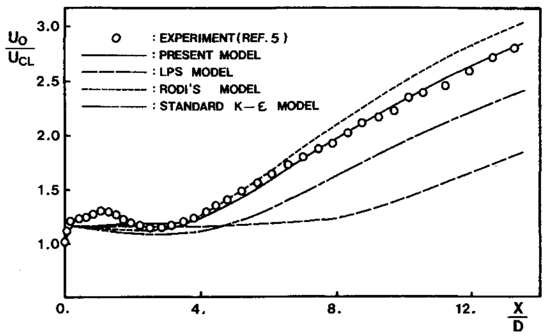

Fig. 1 Comparison of predicted decays of the centerline velocity of a coaxial swirling jet by various models.

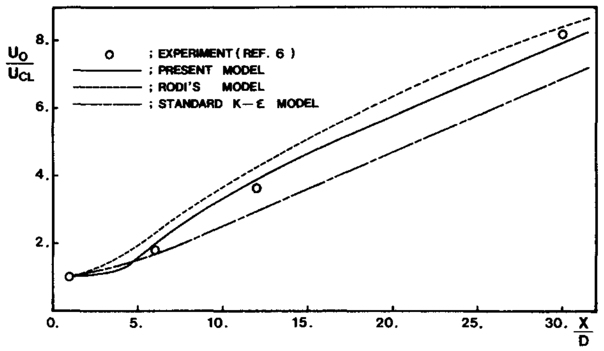

Fig. 2 Comparison of predicted decays of the centerline velocity of a single swirling round jet by various models.

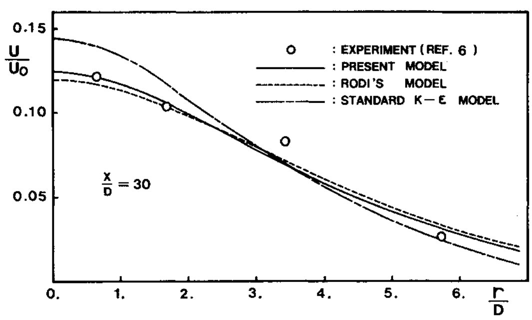

Fig. 3 Comparison of predicted axial velocity profiles of a single swirling round jet.

For a weakly swirling flow without recirculation, it is assumed that

$$\frac{\partial V}{\partial r}=\frac{V}{r}=0,\quad \frac{\partial U}{\partial x}=\frac{\partial V}{\partial x}=\frac{\partial W}{\partial x}=0,\quad \frac{W}{r}\ll\frac{\partial W}{\partial r} \tag{5}$$

These assumptions lead to constitutive equations for Reynolds stresses in cylindrical coordinates as follows:

$$-\overline{uv}=\left[\phi_1\phi_2\Big/\left(1+4\phi_1^2\frac{k^2}{\epsilon^2}\frac{W}{r}\frac{\partial W}{\partial r}\right)\right]\frac{k^2}{\epsilon}\frac{\partial U}{\partial r} \tag{6}$$

$$-\overline{vw}=\left[\phi_1\phi_2\Big/\left(1+4\phi_1^2\frac{k^2}{\epsilon^2}\frac{W}{r}\frac{\partial W}{\partial r}\right)\right]\frac{k^2}{\epsilon}\frac{\partial W}{\partial r} \tag{7}$$

$$-\overline{uw}=\phi_1\frac{k}{\epsilon}\left(\overline{vw}\frac{\partial U}{\partial r}+\overline{uv}\frac{\partial W}{\partial r}\right) \tag{8}$$

For flows without recirculation, the last shear stress $\overline{uw}$ has a negligible effect on the development of the flowfield.[9] Then, eddy viscosities in Eqs. (6) and (7) have the following common expression:

$$\nu_t=\frac{\alpha}{1+\beta Ri}\frac{k^2}{\epsilon} \tag{9}$$

where

$$Ri=\frac{k^2}{\epsilon^2}\frac{W}{r}\frac{\partial W}{\partial r},\qquad \alpha=\phi_1\phi_2,\qquad \beta=4\phi_1^2$$

Under the assumptions of Eq. (5), this Richardson number Ri becomes identical to the turbulent Richardson number defined by Launder et al.[2] for the computation of swirling flows.

In order to match the variable eddy viscosity model Eq. (9) to the constant coefficient model Eq. (1) for the case of a vanishing Ri, α must be equal to $C_\mu(=0.09)$. Since the return-to-isotropy constants c_1 and c_2 have been used in the literature in the ranges $1.5\le c_1\le 1.8$ and $0.5\le c_2\le 0.8$, the new model constant β should be in the range of $0.05\le\beta\le 0.44$ under the local equlibrium assumption $P=\epsilon$. In this work, $\beta=0.25$ is chosen as an average value of this range. In the case of a very weak swirl ($\beta Ri\ll 1.$), $1/(1+\beta Ri)\approx 1-\beta Ri$, and Eq. (9) reduces to a Monin-Oboukhov type that has often been used in the zero-equation model.[10]

Results and Discussion

In the numerical calculation, the choice of initial conditions has a large effect on the final calculations, especially in the free jet case.[11] In the present work, the measured profiles of three velocity components and the turbulent kinetic energy at the jet exit were used as initial conditions. The initial profile of ϵ, which is not available from the experiment, was estimated by the following equation using the experimental profile of $\overline{uv}$[11];

$$\epsilon=C_\mu\frac{k^2}{\overline{uv}}\frac{\partial U}{\partial r}$$

Figure 1 shows nondimensional axial velocity profiles along the jet centerline of a coaxial swirling jet. U_{CL} and U_0 indicate axial velocities along the centerline and at the center of the jet exit respectively, and D is the jet diameter. As has been pointed out by Leschziner and Rodi,[11] the model of Launder, Priddin, and Sharma (LPS model)[2] yields a rather slower decay of the mean centerline velocity compared with that of the standard k-ϵ model. In contrast, Rodi's model[3] yields a faster decay of U_{CL}. It is found that the present model gives the correct decay of the mean centerline velocity. The single swirling round jet in Ref. 6 has been computed by the present model and the Rodi's model, and the results are compared in Figs. 2 and 3. Again, the present model yields the correct decay profile of the centerline velocity, and the axial mean velocity profile at $x/D=30$ is better predicted by the present model as shown in Fig. 3.

Since the model has been derived from the Reynolds stress equation (in addition to this computational accuracy), the present model works well in a wider range of swirling flows than did previous k-ϵ models. Moreover, it is considerably cost-effective compared with the Reynolds stress model.

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Abstracts from Soviet Aerospace Literature

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